Math 53 Discussion Problems Nov 7

1. * Find the maximum value of $f(x,y) = 6xye^{-2x-3y}$ in the closed first quadrant.

2. Find the minimum volume for a region bounded by the planes $x = 0, y = 0, z = 0$ and a plane tangent to the ellipsoid $\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1$.

3. * A heat-seeking particle has the property that at any point (x,y) in the plane it moves in the direction of maximum temperature increase. If the temperature at (x,y) is $T(x,y) = -e^{-2y}\cos x$, find an equation $y = f(x)$ for the path of a heat-seeking particle at the point $(\frac{\pi}{4}, 0)$

4. Let S be the surface that is the graph of $f(x,y) = 10 - x^2 - y^2$. Suppose that the temperature in space at each point (x,y,z) is $T(x,y,z) = x^2y + y^2z + 4x + 14y + z$. Which direction tangential to S at the point $(1,1,8)$ will make the rate of change of temperature a maximum?

5. Let $D_{\mathbf{u}}f$ denote the derivative of $f(x,y) = \frac{x^2+y^2}{2}$ in the direction of the unit vector $\mathbf{u} = u_1\mathbf{i} + u_2\mathbf{j}$. Find the average value of $D_{\mathbf{u}}f$ over the triangular region cut from the first quadrant by the line $x + y = 1$.

6. Evaluate $\int_0^a \int_0^b e^{max\{b^2x^2, a^2y^2\}} dydx$ where a and b are positive numbers and $max\{p,q\}$ denotes the larger of the two numbers p, q.

7. * A parabolic satellite dish is 2m wide and $\frac{1}{2}$m deep. Its axis of symmetry is tilted 30° from the vertical. Set up, but do not evaluate a triple integral in rectangular coordinates that gives the amount of water the satellite dish will hold. (Hint: Put your coordinate system so that the satellite dish is upright and the plane of the water level is slanted)

8. * Evaluate $\int_0^\infty e^{-x^2} dx$ by evaluating $\int_0^\infty \int_0^\infty e^{-x^2-y^2} dxdy$ using polar coordinates.